THERMOMECHANICAL OSCILATIONS IN X-RAY BURSTERS AND RECURRENT NOVAE

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ABSTRACT: A classical and post-newtonian model of X-ray burster and recurrent nova is studied.

A bifurcation point is found. The condition for oscilating behaviour and the corresponding period are computed.

INTRODUCTION

In this work we introduce and study a highly simplified model of X-ray burster or recurrent nova (c.f. [4], [5], [6], [7]), with spherical simmetry, in order to describe the luminosity fluctuations of these astronomical objects. The model consists on a central spherical nucleus of neutronic density (a neutron star) sorrounded by a fermion gas, enclosed in a spherical dust shell, in thermical equilibrium with the gas. The shell emits black body radiation, and the luminosity fluctuations are caused by the oscilation of the shell radius. The gravity forces and the pressure of the radiation force make that the shell can be oscillate if the relevant parameter lies between certain bounds (other wise the shell will collapse into the neutron star or will be ejected); in this case, the internal radiation density it supposed worthless with respect to the external. We shall compute these bounds and show that, in this problem, a primary bifurcation exists when we describe the solutions in terms of the ratio of the shell mass and the gas mass.

2. THE SHELL DINAMICS

We shall suppose that the gas mass is conserved and that the density of the gas is constant, therefore the gas density is:

$$\rho(R) = \frac{3}{4\pi} \cdot \frac{m_g}{R^3}$$

where m_g is the mass of the gas and R the radius of the shell. The density will obey the state equation of a perfect gas $P = \rho . K . T$, where P is the pressure, T the absolute temperature and K the general gas constant. We shall consider that when the shell oscilate the gas undergoes adiabatic evolutions $P = \alpha . \rho^{5/3}$, where α is determined by the initial conditions of the motion.

In order to simplify the equations we introduce the following scale factors, a space scale R_o and a time scale t_o:

$$R_o = \left(\frac{b}{4\pi K^4}\right)^{1/3} . M^{1/3} . \left(\frac{d}{\Omega}\right)^{1/3} . \alpha \qquad (1.a)$$

$$t_o = \frac{1}{2\pi^{1/2}} \cdot \frac{1}{K^2} \cdot \left(\frac{b}{G}\right)^{1/2} \cdot \left(\frac{d}{\Omega}\right)^{1/2} \cdot \alpha^{3/2} \qquad (1.b)$$

where M is the mass of the neutron star, d $=\frac{m_g}{M}$, b is $\frac{4\sigma}{c}$, where σ is the Stefan-Boltzmann constant, and c is the light velocity, and Ω a parameter defined by

$$\Omega = \frac{4}{243} \cdot \frac{b}{K^4} \cdot \frac{G^3M^2}{d} \cdot \left(\frac{m_s}{m_g}\right)^3 \qquad (2)$$

where G is the universal gravitational constant. Using R and t_o as units of lenght and time respectively, the dynamical equation of the shell is:

$$x'' = f(x,x') \qquad (3)$$

where x is the shell radius and the primes are time derivations. Using classical mechanics it is easy to show that

$$f(x,x') = f_o(x,x') = f_o(x) = -\frac{1}{x^2} + \frac{1}{x^3} - \frac{\Omega}{x^6} \qquad (4.a)$$

Also if we use the post-newtonian approximation (cf.eg. |8|) we obtain the corrected function:

$$f(x,x') = f(\delta;x,x') = -\frac{1}{x^2}\left(1-\frac{3}{4}\delta x'^2\right) + \left(\left[\delta + \left(1-\frac{\delta x'^2}{4}\right)^{1/2}\right]\frac{1}{x^3} - \Omega\left(1-\frac{\delta x'^2}{4}\right)\frac{1}{x^6} \qquad (4,b)$$

where

$$\delta = \frac{G}{c^2} \cdot \frac{(4\pi K)^{4/3}}{b^{1/3}} \cdot M^{2/3} \cdot \left(\frac{\Omega}{\alpha}\right)^{1/3} \cdot \frac{1}{\alpha} = \frac{1}{\tilde{c}^2} \qquad (5)$$

where c is the light velocity, and $\tilde{c}$ is the adimensionalized light velocity.

In papers [1] and [2] we showed that the solutions of this equation fits quite well the behaviour of the astronomical objects we study -X-ray bursters and recurrent novae- if we use reasonable physical parameters.

In this paper we are interested in the study of the mathematical properties of eq. (3) in the phase space, and to see how we can obtain a periodic luminosity.

3. PROPERTIES OF THE DYNAMICAL EQUATION

From equations (4.a) and (4.b) we deduce:

Property 1 If $f(\delta=0,x,x') = f_o(x,x')$ then

$$x'' = f_o(x,x') \tag{6}$$

is the classical Newtonian equation of motion of the shell. Also:

Property 2 Via the transformation

$$x \longrightarrow \tilde{x} \quad , \quad \Omega \longrightarrow \tilde{\Omega} \tag{7.a}$$

where

$$\tilde{x} = \frac{1 - 3/4\ \delta x'^2}{\delta + (1 - \frac{\delta x'^2}{4})^{1/2}} \cdot x$$

$$\tilde{\Omega} = \frac{(1 - \delta/4\ x'^2)^{1/2} (1-3/4\ \delta x')^3}{\left[\delta + (1 - \delta/4\ x'^2)^{1/2}\right]^4} \tag{7.b}$$

function $f(\delta;x,x')$ becomes function $f_o(\tilde{x},\tilde{x}')$. Therefore, the singular points of eq. (4.b) can be obtain solving the classical problem, i.e. eq. (4.a). Besides this equation has a simple analitical solution.

Therefore it is easy to verify that:

Property 3 Equation (4.a) has at most two real singular points, a port $x^-(\Omega)$ and a center $x^+(\Omega)$ given by:

$$x^{\pm}(\Omega) = \frac{1 - g(y)}{4} \left[1 \pm (1 - h(y))^{1/2}\right]$$

where

$$g(y) = -(1 + 8y)^{1/2}$$

$$h(y) = 16 \cdot \frac{y}{g(y)\left[g(y)-1\right]}$$

$$y = \left(\frac{\Omega}{16}\right)^{1/3} \cdot \left\{\left[1 + \left(1 - \frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3} + \left[1 - \left(1 - \frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3}\right\}$$

and $\Omega_c = 0.105456$.

Figure 1 shows the real singular points $x^{\pm}$ as a function of Ω. Ω_c is a bifurcation point for Ω, because if $\Omega > \Omega_c$ there are no real singular points and the solution of eq. (4.a) yield a collapse of the shell. Solution $x^+(\Omega)$ corresponds to a center, a stable singular point, which goes to 1 when $\Omega = 0$ (in this case there is no radiation term in eq. (4.a)) $x^+(\Omega)$ lies between 3/4 and 1. (i.e. $x^+(\Omega_c)$ and $x^+(0)$ respectively). On the otherhand $x^-(\Omega)$ corresponds to a port and lies between 0 and 3/4 (i.e. $x^-(0)$ and $x^-(\Omega_c)$ respectively).

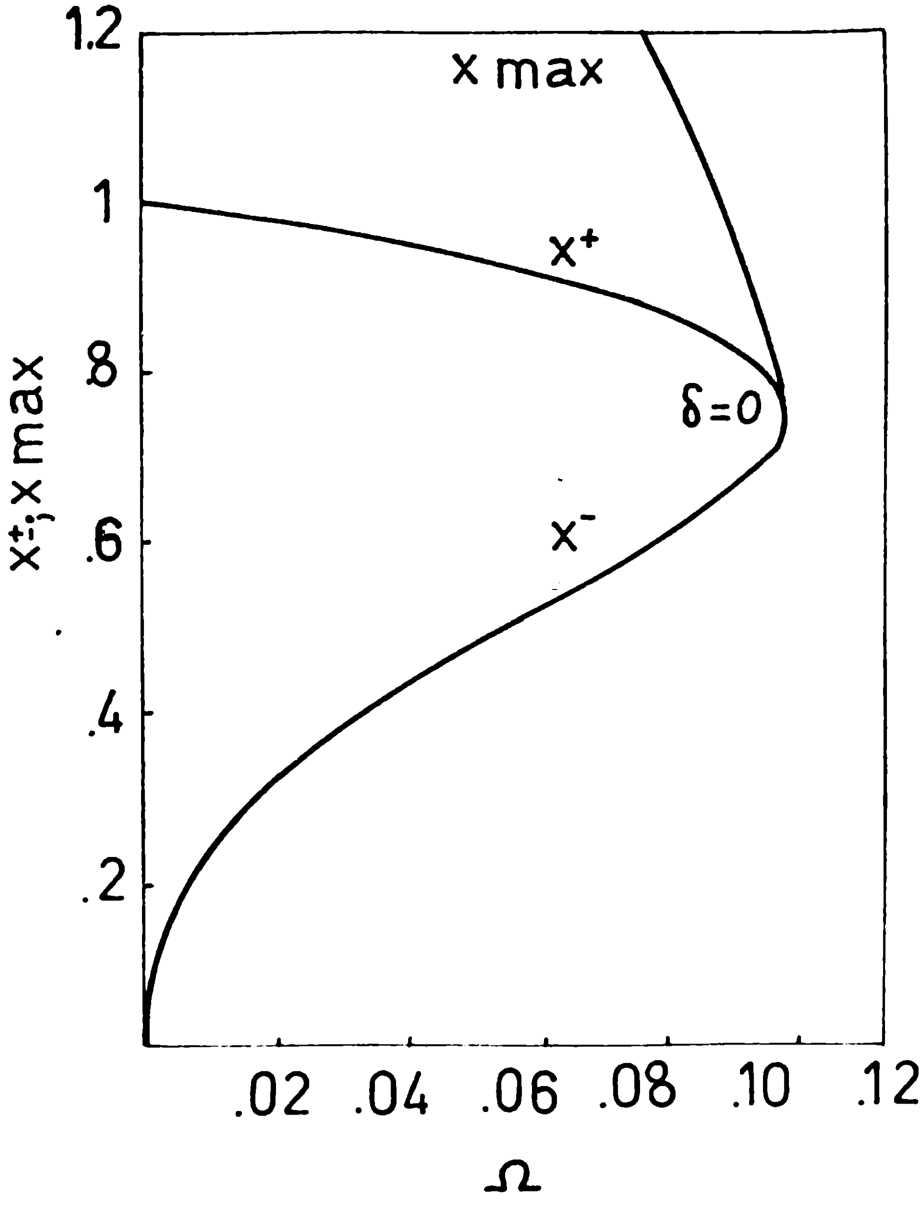

Figure 1: Singular points how Ω function, for $\delta = 0$.

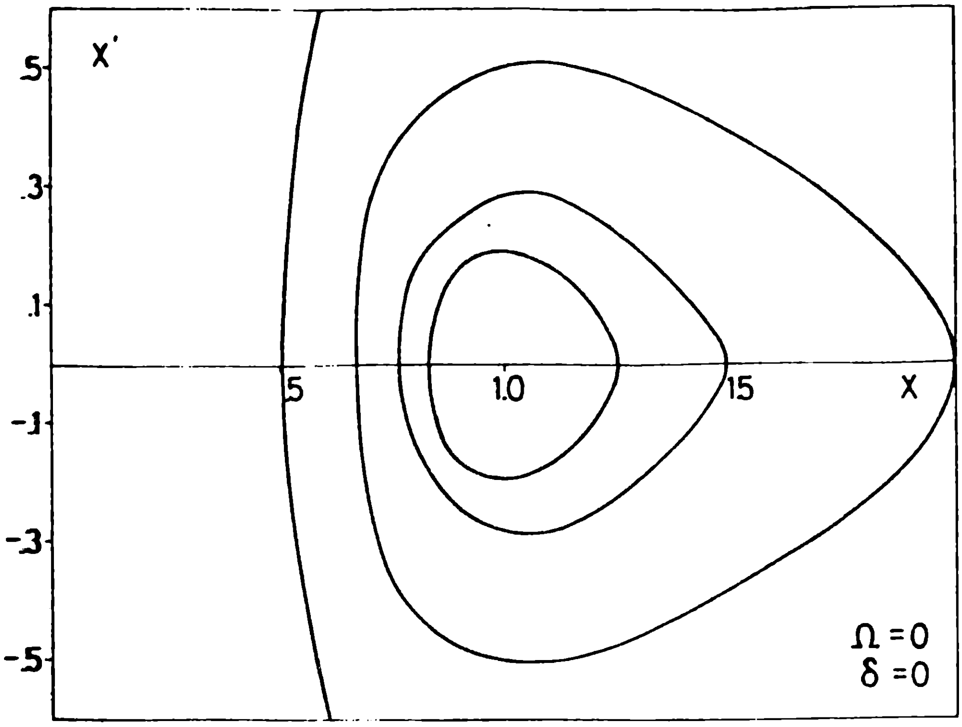

Figure 2.a.: Phase diagram for $\Omega = 0$ (i.e. no radiation). All trayectories are stable oscilation.

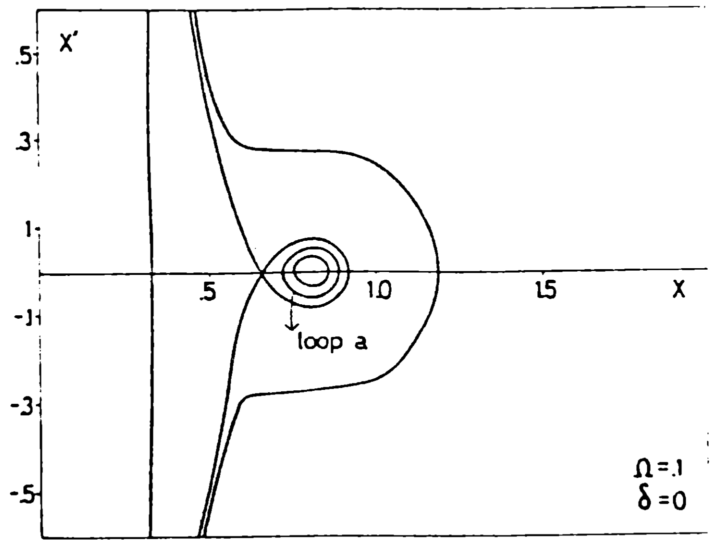

Figure 2.b.: Phase diagram for $\Omega = 0,1$ (i.e. $\Omega < \Omega = 0.10546$). There are two singular points X^+ and X^-. For $0,67 < X < 0,92$ the oscilations may occur. For X outside this interval the shell always collapse.

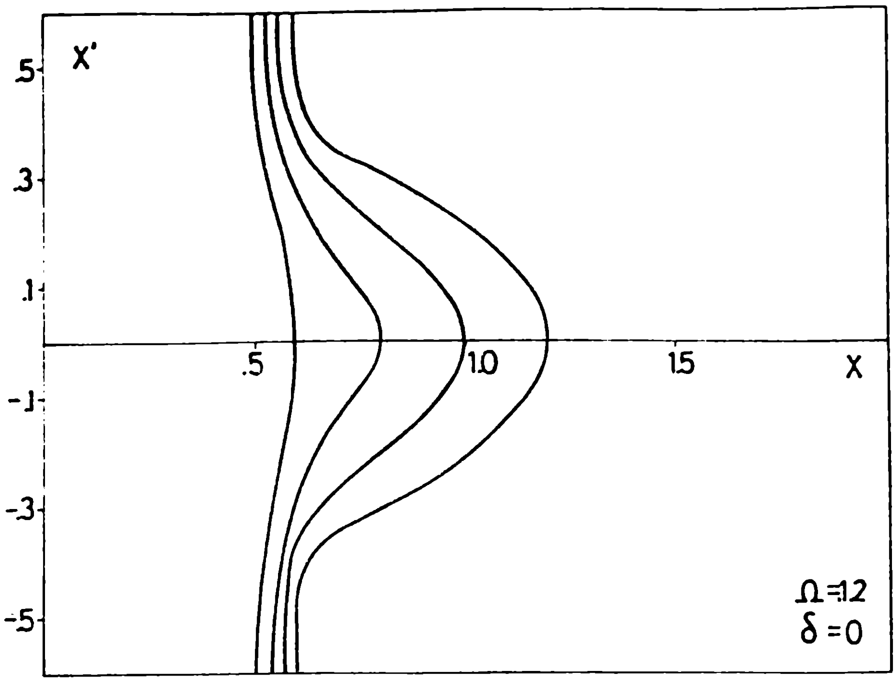

Figure 2.c.: Phase diagram for Ω = 0,12 (i.e. $\Omega > \Omega_C$ = 0,10546). There are no singular points. All trayectories yield the shell collapse.

In figures (2.a), (2,b) and (2,c), we show some orbits for δ = 0 and different values of Ω.

In Figure 3 (break line), we represent the bands where the acceleration keeps its sign i.e.,

- for $\Omega < \Omega_C$ is

$$x'' \begin{cases} >0 & \text{if} \quad x \in \left[x^-(\Omega) ; x^+(\Omega) \right] \\ <0 & \text{if} \quad x < x^-(\Omega) \text{ or } x > x^+(\Omega) \end{cases}$$

- for $\Omega > \Omega_C$ is $x'' < 0 \ \forall \ x$.

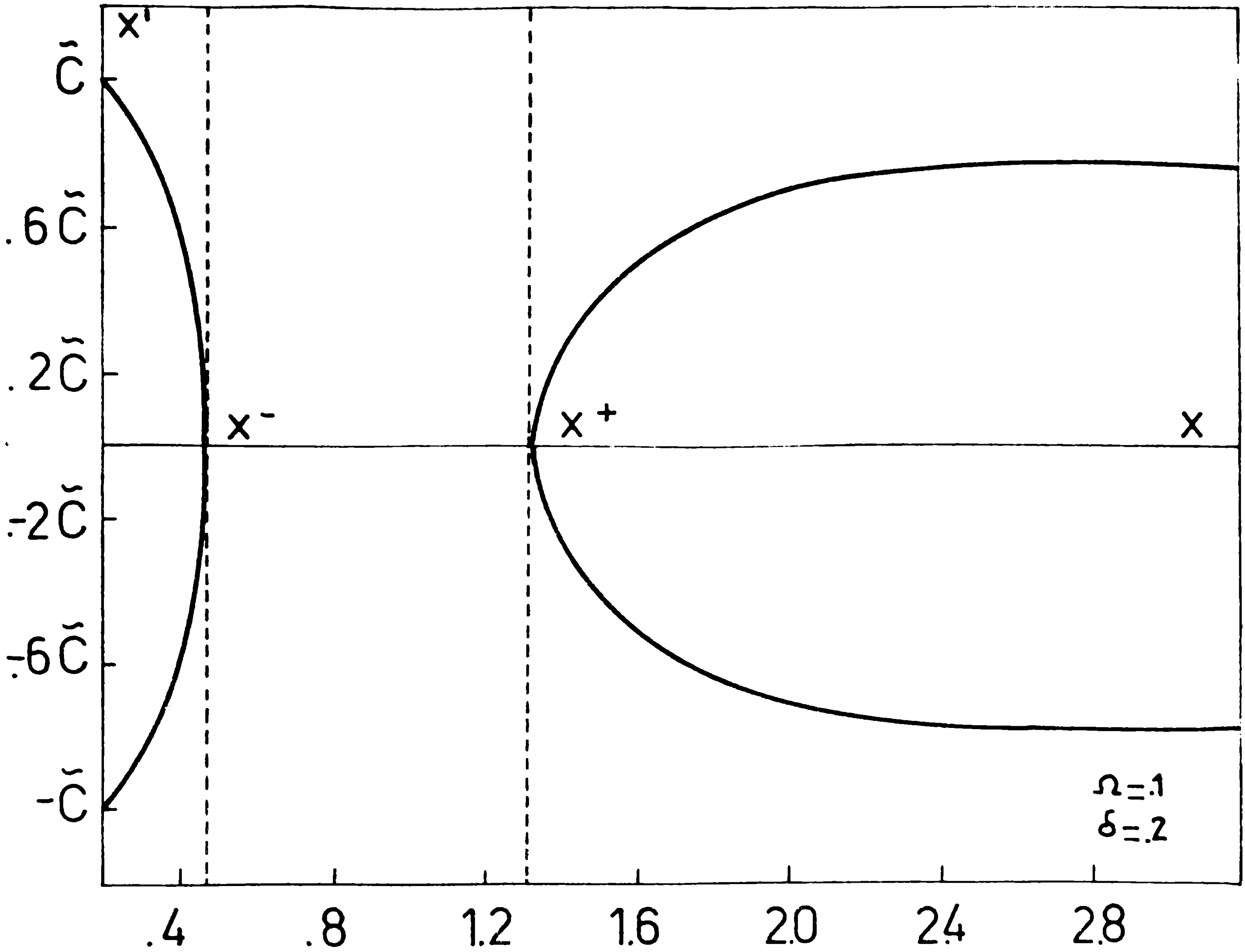

Figure 3: Representation of the bands where the acceleration keeps differents signs.

The bifurcation point, defined by Ω_c, yields an upper bound to the ratio m_g/m_g, if we want that the shell oscilate.

$$(m_s/m_g)^3 \leq \Omega_c \cdot \frac{243}{4} \cdot \frac{K^4}{\pi b} \cdot \frac{d}{G^3M^2}$$

From properties 2 and 3 we obtain,

Property 4 When $\delta \neq 0$ the real singular point of eq. (4.b) are

$$x^{+}(\Omega,\delta) = (1+\delta) . x^{\pm}(\tilde{\Omega})$$

Therefore the bifurcation point is now

$$\Omega_C(\delta) = (1+\delta)^4 . \Omega_C$$

In Figure 4 we represent the curves $x^{\pm}(\Omega, \delta)$ for several values of δ.

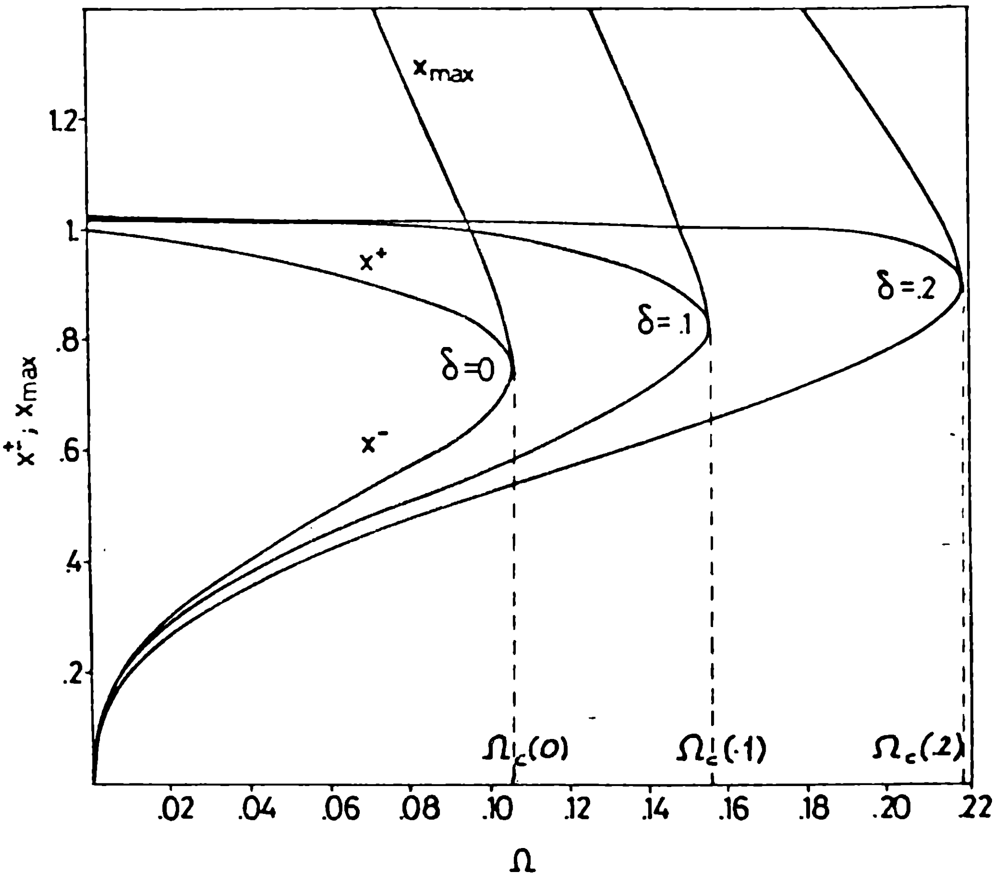

Figure 4: Singular points as a functions of Ω for different δ.

In Figure 3 (complete line) it is shown the phase space for $\Omega < \Omega_C(\delta)$ where the roots of x" are defined by

$$x^{\pm}(x^-,\Omega,\delta) = x^+(\tilde{\Omega}) \quad . \frac{\delta + (1 - \delta/4\, x'^2)^{1/2}}{1 - 3/4\delta\, x'^2}$$

Now we can compute the oscilation period:

Property 5 The oscilation period around the center when δ = 0 is

$$\not{\!p}(\Omega) = \frac{2\pi}{\frac{1}{x^+(\Omega)^7}\left[3x^3 - 2x^4 - 6\Omega\right]_{x=x^+(\Omega)}}$$

For $\delta \neq 0$ if we use transformations (7) the corresponding time transformation is $t \longrightarrow \tilde{t}$ where,

$$\tilde{t} = t . \frac{(1 - 3/4\ \delta x'^2)^{3/2}}{\left[\delta + (1- \ /4x'^2)^{1/2}\right]^{3/2}} \qquad (7.d)$$

from this transformation (bound around of $x^+(\Omega,\delta)$), we can obtain the period for $\delta \neq 0$,

$$\not{\!p}(\Omega,\delta) = \tilde{\not{\!p}}\ (\tilde{\Omega})\ .\ (1 + \delta)^{3/2}$$

In Figure 5 we represent the semiperiods of oscilation as a function of Ω and δ .

This complete the study of our equations.

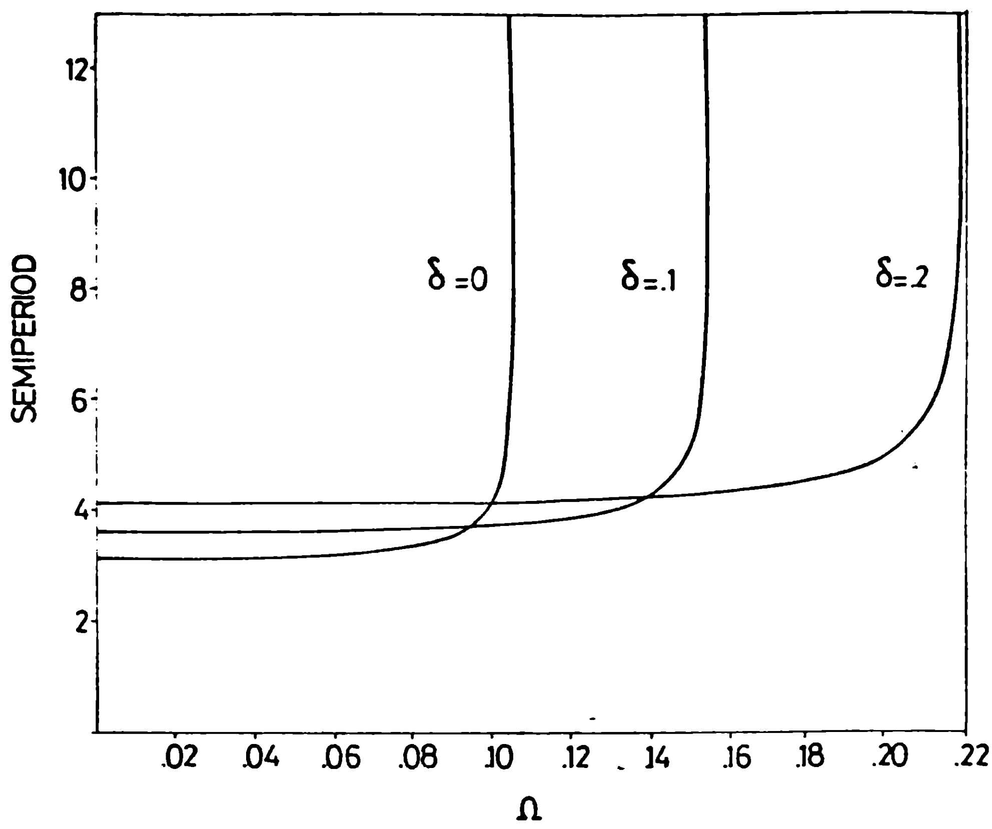

Figure 5: The semiperiod as a functions of Ω for different δ.

4. LUMINOSITY

The luminosity of a star is defined by the energy radiated by unit of time; considering the spherical symmetry of the shell and that the radiation law obeys the vision of a black body the luminosity dimentionless is

$$L(t) = \frac{1}{x^6(t)}$$

being $L(t) = \frac{\mathbb{L}(t)}{\mathbb{L}_0}$ where $\mathbb{L}_0$ is the scale factor defined by

$$\mathbb{L}_0 = \frac{\sigma K^4}{b^2} \cdot (4\pi)^{1/3} \cdot 3^{8/3} \cdot d^{2/3} \cdot M^{2/3} \cdot \frac{\Omega}{\alpha}$$

In the same way we introduce the temperature,

$$T(t) = \frac{\mathbb{T}(t)}{\mathbb{T}_0} = \frac{1}{x^2(t)}$$

where $\mathbb{T}_0$ is the scale factor,

$$\mathbb{T}_0 = \left(\frac{3}{b}\right)^{2/3} \cdot K^{5/3} \cdot \frac{\Omega}{\alpha}^{2/3}$$

In the Figure 6, the periodical fluctuation of luminosity and the shell temperature for classical oscilations around the center have been shown (L(t) and T(t) correspond to "loop a" in Figure (2.b)).

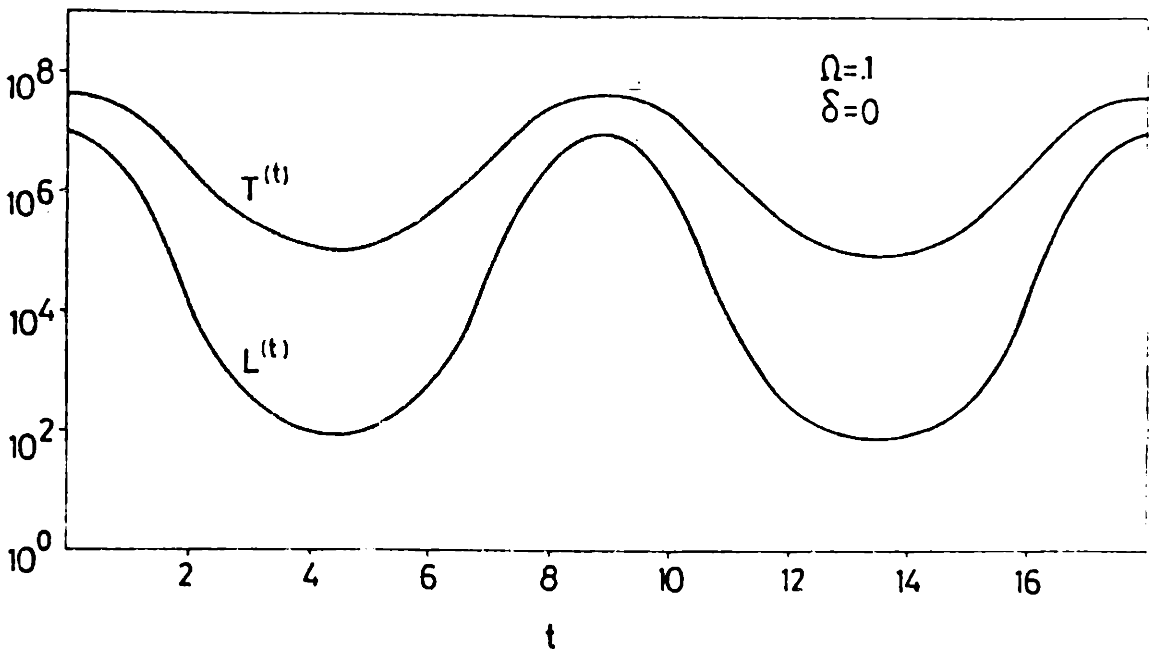

Figure 6: Dependence of the luminosity L in function of time t, and the temperature T with the time t.

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